

ac- and dc-driven noise and I - V characteristics of magnetic nanostructures

O. A. Tretiakov and Aditi Mitra

Department of Physics, New York University, New York, New York 10003, USA

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We study a structure consisting of a ferromagnetic (F) layer coupled to two normal metal (N) leads. The system is driven out of equilibrium by the simultaneous application of external dc and ac voltages across the N/F/N structure. Using the Keldysh diagrammatic approach, and modeling the ferromagnet as a classical spin of size $S \gg 1$, we derive the Langevin equation for the magnetization dynamics and calculate the noise correlator. We find that the noise has an explicit frequency dependence in addition to depending on the characteristics of the ac and dc drive. Further, we calculate the current-voltage characteristics of the structure to $\mathcal{O}(1/S^2)$ and find that the nonequilibrium dynamics of the ferromagnetic layer gives rise to corrections to the current that are both linear and nonlinear in voltage.

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I. INTRODUCTION

Magnetization dynamics in small nanomagnets has recently attracted a lot of theoretical¹⁻⁵ and experimental^{6,7} attention due to advances in manufacturing magnetic nanostructures. The topic of magnetization noise has become an exciting subject owing to its possible influence on magnetization switching² and conductivity of these structures.⁸ It has been shown that the noise in magnetic structures, such as spin valves, can be colored, i.e., it can have a nontrivial frequency dependence.⁹ In diffusive metallic conductors colored noise has been observed experimentally,¹⁰ however in magnetic structures it still requires further investigation.

Experiments involving magnetic nanostructures typically involve the simultaneous application of dc and ac voltage where the ac bias is found to aid the magnetization switching. Therefore in this paper we study a normal metal/ferromagnet/normal metal (N/F/N) structure which has been driven out of equilibrium by the simultaneous application of a dc and ac voltage. We show that the effect of this driving is to produce a noise in the magnetization dynamics that is colored. In addition we determine how the I - V characteristics of the device are affected by the dynamics of the ferromagnetic layer.

The schematic of the N/F/N structure we study is shown in Fig. 1. The ferromagnetic layer is assumed to be small so that it may be modeled as a single-domain magnet. At the same time, the spin of the magnet is considered to be large (spin $S \gg 1$) so that it can be treated as a classical variable. The aim of this paper is twofold, one is to derive the Langevin equation for the magnetization dynamics and second is to present a calculation of the I - V characteristics. In the absence of any magnetization dynamics, the N/F/N structure is Ohmic.¹¹ We show that the dynamics of the ferromagnet gives rise to corrections to the I - V characteristics that are both linear and nonlinear in voltage.

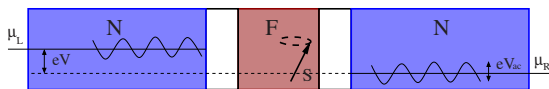


FIG. 1. (Color online) A sketch of the N/F/N structure. The left and right normal-metal leads are coupled through tunnel barriers to the ferromagnetic layer.

The paper is organized as follows. In Sec. II we present the model. In Sec. III we study the nonequilibrium properties of the model in the limit $S \rightarrow \infty$, when the magnetization is static. In Sec. IV we study small fluctuations of the magnetization about the ordering direction thus deriving the Langevin equation and the noise spectrum. The results of this section are then used in Sec. V to calculate the corrections to the current-voltage characteristics arising due to the magnetization fluctuations. Finally in Sec. VI we summarize our results.

II. MODEL

We consider a model Hamiltonian $H = H_m + H_l + H_t$, where H_m describes the ferromagnetic layer, H_l represents the two normal-metal leads, and H_t models the tunneling between the leads and magnetic layer. The Hamiltonian for the magnetic layer H_m is

$$H_m = -(DS_z^2 + BS_z) + J \sum_i \mathbf{S} \cdot \mathbf{s}_i + \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} d_{\mathbf{k}\sigma}^\dagger d_{\mathbf{k}\sigma}. \quad (1)$$

Here the first term models a material (or shape) anisotropy with the anisotropy constant D , the second term describes the interaction of the macrospin \mathbf{S} with the magnetic field \mathbf{B} applied for simplicity in the same z direction as the anisotropy. The third term describes the interaction of the macrospin with the spins of the itinerant electrons \mathbf{s}_i as in the s - d model. It can be rewritten as $J \sum_i (S_z s_{zi} + S_+ s_{-i} + S_- s_{+i})$, where $S_{\pm} = (S_x \pm iS_y)/2$, $s_{\pm i} = s_{xi} \pm is_{yi}$, and $\mathbf{s}_i = \frac{1}{2} \sum_{\mathbf{k}, \sigma, \alpha, \beta} d_{\mathbf{k}\sigma, \alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} d_{\mathbf{k}\sigma, \beta}$ with $\sigma_{\alpha\beta}$ being the components of Pauli matrices. Here $d_{\mathbf{k}\sigma}^\dagger$ ($d_{\mathbf{k}\sigma}$) creates (destroys) an electron in state with momentum \mathbf{k} and spin component σ . The pure macrospin part of the Hamiltonian can be rewritten as $-(DS_z^2 + BS_z) \approx \text{const} + bS_+ S_-$, where the constant part is $-DS^2 - BS$ and $b = 4[D + B/(2S)]$. Nanomagnets are typically characterized by a significant anisotropy. This along with the fact that $S \gg 1$ implies that the fluctuations of the nanomagnet about the ordering direction are small. Our theoretical treatment will therefore involve a perturbative expansion in spin fluctuations, which as we shall show is equivalent to an expansion in $1/S$.

We assume that the electrons in the leads are noninteracting. To model the ac bias voltage we introduce a time dependence of the lead single-particle energies,¹² namely, $\epsilon_{k\alpha}(t) = \epsilon_{k\alpha}^0 + V_{ac} \cos(\omega_0 t + \varphi_\alpha)$ where α labels the left (L) or right (R) lead, and V_{ac}, ω_0 are, respectively, the amplitude and frequency of the ac bias. Thus, the lead Hamiltonian is

$$H_l = \sum_{k\sigma, \alpha \in L, R} \epsilon_{k\alpha}(t) c_{k\sigma\alpha}^\dagger c_{k\sigma\alpha}. \quad (2)$$

The coupling between the leads and magnetic layer is

$$H_t = \sum_{\mathbf{k}, k_0, \sigma, \alpha \in L, R} (t_\alpha c_{k\sigma\alpha}^\dagger d_{\mathbf{k}\sigma} + \text{H.c.}). \quad (3)$$

In Eq. (2) $k = (\mathbf{k}, k_0)$ where \mathbf{k} is a two-dimensional momentum in the plane perpendicular to the tunneling direction and is assumed to be conserved on tunneling.

To study this nonequilibrium problem we employ the Keldysh formalism.^{3,13} We introduce variables $\mathbf{S}^{cl} = (\mathbf{S}^+ + \mathbf{S}^-)/2$ and $\mathbf{S}^q = (\mathbf{S}^- - \mathbf{S}^+)/2$ where the upper \pm indices correspond to the time-ordered (antitime-ordered) directions on the Keldysh contour, and the Keldysh path integral takes the form, $\mathcal{Z}_K = \int \mathcal{D}[\mathbf{S}^{cl}, \mathbf{S}^q] e^{-iS_K}$. Here S_K is the effective action for the macrospin obtained formally by integrating out all fermionic degrees of freedom

$$S_K = 2b \text{Tr}(S_+^{cl} S_-^q + S_+^q S_-^{cl}) + i \text{Tr} \ln \left[\hat{g}_{d\sigma}^{-1} - \hat{\Sigma} - J \hat{S}_z \frac{\sigma_z}{2} - J \hat{S}_+ \sigma_- - J \hat{S}_- \sigma_+ \right], \quad (4)$$

where $\hat{S}_{a=z, \pm} = \begin{pmatrix} S_a^{cl} & S_a^q \\ S_a^q & S_a^{cl} \end{pmatrix}$ and $\hat{g}_{d\sigma} = \begin{pmatrix} g_{d\sigma}^R & g_{d\sigma}^K \\ 0 & g_{d\sigma}^A \end{pmatrix}$ are the Green's functions of the free electrons in the magnetic layer, $\sigma_x, \sigma_y, \sigma_z = (\sigma_x \pm \sigma_y)/2$ are Pauli matrices, and $\hat{\Sigma} = \begin{pmatrix} \Sigma^R & \Sigma^K \\ 0 & \Sigma^A \end{pmatrix}$ is the self-energy due to coupling to the leads. As we shall show, $\hat{\Sigma}$ depends on the ac and dc bias and is independent of σ because the leads are nonmagnetic. In what follows we make the assumption that the fluctuations of the macrospin from the ordering direction are small. Thus we write $J \hat{S}_z = JS + J \delta \hat{S}_z$ and eventually expand Eq. (4) perturbatively in the fluctuations $\delta \hat{S}_z, \hat{S}_\pm$.

III. GREEN'S FUNCTIONS IN THE MAGNETIC LAYER

We first discuss the properties of the nonequilibrium system when the magnetization does not fluctuate. Denoting G_0 to be the Green's function of the electrons in the magnetic layer when $\delta \hat{S}_z = \hat{S}_\pm = 0$, Eq. (4) implies

$$[G_{0\sigma}^R]^{-1} = [g_{d\sigma}^R]^{-1} - \Sigma^R - JS \frac{\sigma}{2}, \quad (5a)$$

$$G_{0\sigma}^K = G_{0\sigma}^R \circ \Sigma^K \circ G_{0\sigma}^A. \quad (5b)$$

The symbol \circ in Eq. (5) denotes convolution in the time domain and the self-energies due to coupling to leads are

$$\Sigma^{R(K)}(t, t') = \sum_{k_0, \alpha} t_\alpha^2 g_{k\sigma\alpha}^{R(K)}(t, t'). \quad (6)$$

$g_{k\sigma\alpha}^{R(K)}$ are the retarded and Keldysh components of the electron Green's function in the leads and are defined as

$$g_{k\sigma\alpha}^R(t, t') = -i \theta(t - t') \langle \{c_{k\sigma\alpha}(t), c_{k\sigma\alpha}^\dagger(t')\} \rangle, \quad (7a)$$

$$g_{k\sigma\alpha}^K(t, t') = -i \langle [c_{k\sigma\alpha}(t), c_{k\sigma\alpha}^\dagger(t')] \rangle. \quad (7b)$$

Since $c_{k\sigma\alpha}(t) = e^{-i\int_{-\infty}^t dt_1 \epsilon_{k\alpha}(t_1)} c_{k\sigma\alpha}(-\infty)$, we find

$$g_{k\sigma\alpha}^R(t, t') = -i \theta(t - t') e^{-i\int_{t'}^t dt_1 \epsilon_{k\alpha}(t_1)}, \quad (8a)$$

$$g_{k\sigma\alpha}^K(t, t') = -i [1 - 2f(\epsilon_{k\alpha}^0 - \mu_\alpha)] e^{-i\int_{t'}^t dt_1 \epsilon_{k\alpha}(t_1)}, \quad (8b)$$

where f is the Fermi-distribution function in the leads and we have used that $\langle [c_{k\sigma\alpha}(-\infty), c_{k\sigma\alpha}^\dagger(-\infty)] \rangle = 1 - 2f(\epsilon_{k\alpha}^0 - \mu_\alpha)$, where μ_α is the chemical potential of lead α and a dc bias corresponds to $\mu_L \neq \mu_R$. Using the identity $e^{z(a-1)/2} = \sum_{n=-\infty}^{\infty} a^n J_n(z)$, where $J_n(z)$ are Bessel functions of the first kind, we find

$$g_{k\sigma\alpha}^R(t, t') = -i \theta(t - t') e^{-i\epsilon_{k_0, \mathbf{k}\alpha}^0(t-t')} \times \left[\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{V_{ac}}{\omega_0}\right) e^{-in\omega_0(t-t')} + \sum_{n \neq m} J_n\left(\frac{V_{ac}}{\omega_0}\right) J_m\left(\frac{V_{ac}}{\omega_0}\right) e^{i\varphi_\alpha(m-n) - i\omega_0(mt-nt)} \right]. \quad (9)$$

Changing variables to $\tau = t - t'$ and $T = (t + t')/2$ one may write $mt' - nt = (n - m)T + (n + m)\tau/2$. In what follows we average Green's functions over time $T \gg \omega_0^{-1}$, where the averaging is denoted by $\overline{g_{k\alpha}(t, t')}$. This is justified when the magnetization dynamics is slow compared to ω_0^{-1} (a precise condition for this will be given in Sec. IV).

As a result of the time averaging, the terms corresponding to $n \neq m$ in Eq. (9) vanish. This leads to a time-averaged retarded self-energy $\overline{\Sigma}^R(\omega) = -i \sum_{\alpha \in L, R} \Gamma_\alpha$ where $\Gamma_\alpha = \pi \nu t_\alpha^2$ is the decay rate of the electrons into the leads, and ν is the density of states in the leads. In what follows we will assume Γ_α to be independent of energy. From Eq. (6), the time-averaged Keldysh component of the self-energy becomes

$$\overline{\Sigma}^K(\omega) = -2i \sum_{\alpha} \Gamma_\alpha \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{V_{ac}}{\omega_0}\right) [1 - 2f(\omega - n\omega_0 - \mu_\alpha)]. \quad (10)$$

The above discussion implies that the spectral function of electrons in the magnetic layer $A_{0\sigma} = -\text{Im}[G_{0\sigma}^R]$ is the same as in equilibrium, $A_{0\sigma}(\mathbf{k}, \omega) = \Gamma / [(\omega - \epsilon_{\mathbf{k}}^d - \sigma\Delta)^2 + \Gamma^2]$, where $\Gamma = \Gamma_L + \Gamma_R$ and the exchange splitting $\Delta = JS/2$. We will assume $\Delta > \Gamma$ so that the ferromagnetism of the conduction electrons is well defined. Further, the nonequilibrium distribution function f_{neq} of the electrons in the magnetic layer, defined as $G_{0\sigma}^K(\mathbf{k}, \omega) = -2i A_{0\sigma}(\mathbf{k}, \omega) [1 - 2f_{neq}(\omega)]$, is

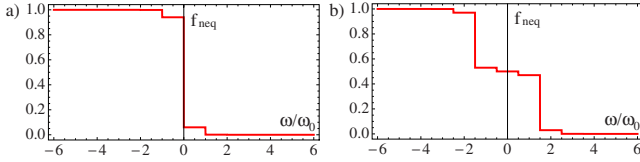


FIG. 2. (Color online) Distribution function of electrons in the magnetic layer $f_{neq}(\omega)$ given by Eq. (11): (a) pure ac voltage case, $\mu_L = \mu_R = 0$ and $V_{ac}/\omega_0 = 0.5$; (b) the case of nonzero ac and dc voltages, the parameters are $\Gamma_L = \Gamma_R$, $\mu_L = -\mu_R = 1.5\omega_0$, and $V_{ac}/\omega_0 = 0.5$. Frequency ω is measured in units of ω_0 .

$$f_{neq}(\omega) = \sum_{\alpha} \frac{\Gamma_{\alpha}}{\Gamma} \sum_n J_n^2\left(\frac{V_{ac}}{\omega_0}\right) f(\omega - n\omega_0 - \mu_{\alpha}), \quad (11)$$

Below we consider the case of zero temperature when the Fermi function $f(\omega) = \theta(-\omega)$. A typical $f_{neq}(\omega)$ is plotted in Fig. 2. While in the pure dc case $f_{neq}(\omega)$ is a weighted sum of Fermi functions of the left and right leads, the ac bias adds steps to f_{neq} at frequencies $\pm |n|\omega_0$ corresponding to photon absorption and emission.

IV. LANGEVIN EQUATION

We expand the effective action, Eq. (4), to quadratic order in the fluctuations to obtain

$$S_K = 2b \text{Tr}(S_+^{cl} S_-^q + S_+^q S_-^{cl}) + (S_z^{cl} \Pi_{zz}^R S_z^q + \text{H.c.}) + S_z^q \Pi_{zz}^K S_z^q + \frac{1}{2} \sum_{\alpha, \beta=x,y} [S_{\alpha}^q \Pi_{\alpha\beta}^K S_{\beta}^q + (S_{\alpha}^{cl} \Pi_{\alpha\beta}^R S_{\beta}^q + \text{H.c.})], \quad (12)$$

where $\Pi_{\alpha\beta}^R$ are the components of the polarization operator that are calculated following standard techniques.³ Note that $\Pi_{xx}(\omega) = \Pi_{yy}(\omega)$ and $\Pi_{xy}(\omega) = -\Pi_{yx}(\omega)$. Moreover, to leading order in spin fluctuations Π_{zz} does not play a role. For small frequencies $\omega \ll \Delta$, we find

$$\Pi_{xx}^R(\omega) \simeq -i\beta_{xx}\omega, \quad \Pi_{xy}^R(\omega) \simeq -i\beta_{xy}\omega, \quad (13a)$$

$$\beta_{xx} = J^2 \nu \frac{\Gamma}{\Delta^2 + \Gamma^2}, \quad \beta_{xy} = J^2 \nu \frac{\Delta}{\Delta^2 + \Gamma^2}. \quad (13b)$$

Note that $\beta_{xy} = \frac{\Delta}{\Gamma} \beta_{xx}$.

The action, Eq. (12), may be diagonalized in the basis $S_{\pm}^{cl} = (S_x^{cl} \pm iS_y^{cl})/2$ thus yielding the Langevin equation³

$$bS_{\pm}^{cl} + (\beta_{xx} \mp i\beta_{xy}) \dot{S}_{\pm}^{cl} = \xi_{\pm}, \quad (14)$$

where $\xi_{\pm} = (\xi_x \pm i\xi_y)/2$ is an auxiliary field representing noise whose correlator is given by $\langle \xi_{a=x,y}(\omega) \xi_{b=x,y}(-\omega) \rangle = i\Pi_{ab}^K(\omega)$. We have found an analytical expression for Π^K in terms of a double sum over squares of Bessel functions. For $V_{ac} \ll \omega_0$ we may keep only terms corresponding to single-photon absorption and emission processes. For $\mu_L = -\mu_R = V/2$ and $\omega \ll \Delta$ the noise correlator is

$$\begin{aligned} \langle \xi_x(\omega) \xi_x(-\omega) \rangle &= i\Pi_{xx}^K(\omega) \simeq 2\beta_{xx} \left\{ \frac{\Gamma_L \Gamma_R}{\Gamma^2} (|\omega + V| + |\omega - V|) \right. \\ &+ \frac{\Gamma_L^2 + \Gamma_R^2}{\Gamma^2} |\omega| + \left(\frac{V_{ac}}{2\omega_0} \right)^2 \sum_{j=\pm 1} \left[\frac{\Gamma_L^2 + \Gamma_R^2}{\Gamma^2} |\omega \right. \\ &+ j\omega_0| + \frac{\Gamma_L \Gamma_R}{\Gamma^2} (|\omega \\ &+ j\omega_0 + V| + |\omega + j\omega_0 - V|) \left. \left. \right] \right\}. \quad (15) \end{aligned}$$

Similarly, the off-diagonal component $\langle \xi_x(\omega) \xi_y(-\omega) \rangle = i\Pi_{xy}^K(\omega)$ is

$$\begin{aligned} \Pi_{xy}^K(\omega) &\simeq J^2 \nu \frac{\Delta \Gamma \omega}{(\Delta^2 + \Gamma^2)^2} \sum_{n,m} J_n^2\left(\frac{V_{ac}}{\omega_0}\right) J_m^2\left(\frac{V_{ac}}{\omega_0}\right) \\ &\times \sum_{\alpha, \beta=L,R} \frac{\Gamma_{\alpha} \Gamma_{\beta}}{\Gamma^2} |\omega + (m-n)\omega_0 + \mu_{\alpha} - \mu_{\beta}|. \quad (16) \end{aligned}$$

An effective temperature T_{eff} may be extracted from the zero-frequency limit of the noise correlator. Equation (15) implies $T_{\text{eff}} \sim (\Gamma_L \Gamma_R / \Gamma^2) V + (V_{ac}/2\omega_0)^2 [\omega_0 + (2\Gamma_L \Gamma_R / \Gamma^2) (V - \omega_0) \theta(V - \omega_0)]$ and therefore has a discontinuity¹⁴ at $V = \omega_0$. In the opposite limit of $\omega \gg \Delta$ the noise correlator vanishes as $\sim 1/\omega$ as expected.

Equation (14) can be rewritten in the form of a stochastic Landau-Lifshitz-Gilbert equation¹⁵

$$\dot{\mathbf{S}} = \gamma \mathbf{S} \times \mathbf{H}_{\text{eff}} - \alpha_0 \mathbf{S} \times \dot{\mathbf{S}} + \xi', \quad (17)$$

where γ is the gyromagnetic ratio, $\mathbf{H}_{\text{eff}} = b\hat{\mathbf{z}}/(\gamma\beta_{xy})$ is an effective magnetic field, the noise is $\xi' = \frac{1}{\beta_{xy}} \hat{\mathbf{z}} \times \xi$, and the Gilbert damping constant is $\alpha_0 = \beta_{xx}/\beta_{xy} = \Gamma/\Delta$.

In order to determine how the magnetization dynamics affects the I - V characteristics we will need the spin response and correlation functions. From Eq. (12) the spin-spin response function is

$$D_{-+}^R(\omega) = i \langle S_-^{cl}(\omega) S_+^q(-\omega) \rangle = \frac{1}{b - \beta_{xy}\omega - i\beta_{xx}\omega} \quad (18)$$

whereas the spin-spin correlation function is

$$D_{-+}^K(\omega) = i \langle S_-^{cl}(\omega) S_+^{cl}(-\omega) \rangle = - \frac{(\Pi_{xx}^K - i\Pi_{xy}^K)(\omega)}{|b - \beta_{xy}\omega - i\beta_{xx}\omega|^2} \quad (19)$$

and $D_{+-}^{R(K)}(\Delta) = D_{-+}^{R(K)}(-\Delta)$. As expected in equilibrium ($V = V_{ac} = 0$) the components of both $D(\omega)$ and $\Pi(\omega)$ satisfy the fluctuation-dissipation theorem.

It is instructive to take the inverse Fourier transform of Eq. (19) to obtain the time dependence of the transverse spin-spin correlation function

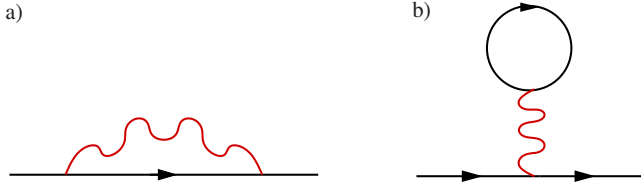


FIG. 3. (Color online) The diagrams for one-loop corrections to the electron Green's function in the magnetic layer due to coupling to spin fluctuations: (a) exchange (Fock) contribution $\Sigma_{\sigma d}^{eR}$ and (b) Hartree contribution $\Sigma_{\sigma d}^{hR}$. Wavy lines correspond to spin-spin correlators $\langle S_{-}(\omega)S_{+}(-\omega) \rangle$.

$$\langle S_{-}^{cl}(t > 0)S_{+}^{cl}(0) \rangle = \frac{(i\Pi_{xx}^K + \Pi_{xy}^K)(\omega_1)}{2b\beta_{xx}} e^{-t/\tau - it/\tau_1}, \quad (20)$$

where $\omega_1 = b(\beta_{xy} - i\beta_{xx})/(\beta_{xy}^2 + \beta_{xx}^2)$, $\tau = \nu J^2/(b\Gamma)$, and $\tau_1 = 2\nu J/(bS)$. Equation (20) shows that the spin-spin correlations decay with the characteristic time τ . Thus as long as $1/\omega_0 \ll \tau$, the macrospin dynamics is rather slow and we can use the time-averaging procedure for the Green's functions outlined in Sec. III.

V. CURRENT-VOLTAGE CHARACTERISTICS

We will now study how the current-voltage characteristics of the magnetic junction are affected by the magnetization dynamics of the ferromagnetic layer. We employ the Jauho-Meir-Wingreen formula¹² for the tunneling current

$$I = \frac{e}{\hbar} \int \frac{d\Omega}{2\pi} \sum_{\mathbf{k}, \sigma} [f(\Omega - \mu_L) - f(\Omega - \mu_R)] \frac{4\Gamma_L \Gamma_R}{\Gamma} A(\mathbf{k}, \Omega). \quad (21)$$

In the following we calculate the leading correction to the spectral function $A_{\sigma} = -\text{Im}[G_{\sigma d}^R]$ due to coupling to spin fluctuations. The spectral function is determined from the Dyson equation $[G_{\sigma d}^R]^{-1} = [G_{\sigma d}^0]^{-1} - \Sigma_{\sigma d}^R$, where $\Sigma_{\sigma d}^R$ is the self-energy due to coupling to spin fluctuations. To one-loop order $\Sigma_{\sigma d}^R = \Sigma_{\sigma d}^{eR} + \Sigma_{\sigma d}^{hR}$, where $\Sigma_{\sigma d}^{eR}$ and $\Sigma_{\sigma d}^{hR}$ are, respectively, the exchange and Hartree contributions to the self-energy, see Fig. 3. To leading order in the fluctuations, it suffices to do perturbation theory in J^2 so that $G_{d\sigma}^R = G_{0\sigma}^R + \delta G_{d\sigma}^R$ with

$$\delta G_{d\sigma}^R = G_{0\sigma}^R \Sigma_{\sigma d}^R G_{0\sigma}^R. \quad (22)$$

Note that $\Sigma_{\uparrow d}$ and $\Sigma_{\downarrow d}$ are related by $\Delta \leftrightarrow -\Delta$.

The exchange contribution to the self-energy is

$$\Sigma_{\downarrow d}^{eR}(\mathbf{k}, \Omega) = -\frac{iJ^2}{2} \int \frac{d\omega}{2\pi} [G_{d\uparrow}^R(\mathbf{k}, \omega + \Omega) D_{-\uparrow}^K(\omega) + G_{d\uparrow}^K(\mathbf{k}, \omega + \Omega) D_{-\uparrow}^A(\omega)]. \quad (23)$$

Keeping terms to leading order in J^2 , Σ^{eR} is purely real and given by

$$\Sigma_{\downarrow d}^{eR}(\mathbf{k}, \Omega) = -\frac{J^2}{\pi b} \sum_{\alpha} \frac{\Gamma_{\alpha}}{\Gamma} \left[\frac{\pi}{2} + \arctan \frac{\epsilon_{\mathbf{k}}^d + \Delta - \mu_{\alpha}}{\Gamma} + \left(\frac{V_{ac}}{2\omega_0} \right)^2 \left(\pi + \sum_{m=\pm 1} \arctan \frac{\epsilon_{\mathbf{k}}^d + \Delta + m\omega_0 - \mu_{\alpha}}{\Gamma} \right) \right]. \quad (24)$$

The Hartree contribution to the self-energy is given by

$$\Sigma_{\downarrow d}^{hR}(\mathbf{k}, \Omega) = -\frac{iJ^2}{2} D_{+-}^A(\omega=0) \int \nu d\epsilon' \int \frac{d\omega'}{2\pi} G_{d\uparrow}^K(\epsilon', \omega')$$

where $D_{+-}^A(\omega=0) = 1/b$ and we have set $\Sigma_{\mathbf{k}'} \rightarrow \int \nu d\epsilon'$. Note that the Hartree contribution is independent of external frequency and momentum, and therefore only shifts the position of the pole of $G_{d\uparrow}^R$ but does not contribute to the corrections to the current.

We denote the total current averaged over time $T \gg 1/\omega_0$ as $\bar{I} = I_0 + \delta \bar{I}$ where

$$I_0 = 4 \frac{e}{\hbar} \frac{\nu \Gamma_L \Gamma_R}{\Gamma} V \quad (25)$$

is the current for a static ferromagnet while $\delta \bar{I}$ is the leading correction due to spin fluctuations computed from Eqs. (21), (22), and (24) for $\mu_L = -\mu_R = V/2$

$$\frac{\delta \bar{I}}{I_0} \simeq \frac{\Gamma J^2}{2\pi b \Delta^2 \left(1 + \frac{\Gamma^2}{\Delta^2}\right)} \left\{ \left(1 + \frac{V_{ac}^2}{2\omega_0^2}\right) \left[1 + \frac{(3\Delta^2 - \Gamma^2)V^2}{12(\Delta^2 + \Gamma^2)^2}\right] + \frac{V_{ac}^2}{16} \left[\frac{3\Delta^2 - \Gamma^2}{(\Delta^2 + \Gamma^2)^2} + \frac{(\sqrt{5}\Delta^2 - \Gamma^2)^2 V^2}{2(\Delta^2 + \Gamma^2)^4} \right] \right\}. \quad (26)$$

Since $\Delta \sim JS$, this correction to the current is $\mathcal{O}(\Gamma/bS^2)$. Thus our perturbative treatment in spin fluctuations is valid as long as $b \neq 0$ and $S \gg 1$. Moreover the correction $\delta \bar{I} > 0$. This is because scattering off spin fluctuations in this geometry produces additional channels for electron conduction (in contrast to a bulk geometry where this scattering would cause the conductivity to decrease). It is worth mentioning that ac bias contributes to the Ohmic corrections as well with terms such as $\sim (V_{ac}^2/\Delta^2)V$ and $(V_{ac}^2/\omega_0^2)V$. For the pure dc case ($V_{ac}=0$), we find that the differential conductance $g = e \partial I / \partial V$ for $\Delta \gg \Gamma$ is

$$g = 4 \frac{e^2}{\hbar} \frac{\nu \Gamma_L \Gamma_R}{\Gamma} \left[1 + \frac{J^2 \Gamma}{2\pi b \Delta^2} \left(1 + \frac{3V^2}{4\Delta^2} \right) \right]. \quad (27)$$

Note that the quadratic in voltage corrections in Eq. (27) are similar in spirit to temperature corrections ($\sim T^2$) to the conductance. Also, our result is for a particular choice of chemical potentials $\mu_L = -\mu_R = V/2$. The answer, in general, will change if a different choice, such as $\mu_L = V$ and $\mu_R = 0$, were used. The reason for this difference is that the symmetric combination of chemical potentials $(\mu_L + \mu_R)/2$ plays the role of a mean chemical potential for the electrons in the nanomagnet, tuning which modifies the equilibrium spectral density and hence the linear-response conductance as well as other equilibrium properties. In an experiment this mean

chemical potential may be tuned by an external gate voltage. Our purely antisymmetric combination $\mu_L = -\mu_R$ avoids these intrinsically equilibrium effects.

VI. SUMMARY

We have derived a Langevin equation for the magnetization dynamics for a simultaneously applied ac and dc bias across an N/F/N nanostructure. The magnetization dynamics is characterized by a frequency-dependent noise, Eq. (15). We have also computed corrections to the I - V characteristics to leading ($1/S^2$) order in the spin fluctuations. These fluctuations are found to not only modify the Ohmic part of the

I - V characteristics but to also give rise to corrections that are nonlinear in voltage, Eq. (26). Experiments often exhibit nonlinear I - V curves⁶ but the origin of the nonlinearities is usually not clear. The usefulness of our result is that the current is a function of three independent experimentally tunable parameters (the dc bias V , the ac amplitude V_{ac} and frequency ω_0) which can in principle allow one to extract the physics arising only from magnetization dynamics.

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